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A Versatile Modeling Method in Electromechanics – The Canonical Equivalent Circuit Representation of a Clamped-free Piezoceramic Multilayered Bending Transducer

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Graphical Abstract

$$\frac{d^4\underline{v}(x)}{dx^4}-\omega^2\frac{\mu}{C}\underline{v}(x)+j\frac{\omega}{C}r_a\underline{v}(x)=j\frac{\omega}{C}\underline{f}(x) \quad (1)$$

$$\underline{v}(x)=\sum_{m=1}^{\infty}X_m(x)\underline{v}_m \quad (2)$$

$$\underline{v}_m\left[\frac{(k_m l)^4}{j\omega n_0}+j\omega m+r\right]=\frac{l\int_0^l \underline{f}(x)X_m dx}{\int_0^l X_m^2 dx} \quad (3)$$

$$\begin{pmatrix}\underline{v}_1\\ \underline{\Omega}_1\\ \underline{v}_2\\ \underline{\Omega}_2\end{pmatrix}=\underbrace{\begin{pmatrix}\underline{h}_{11} & \underline{h}_{12} & \underline{h}_{13} & \underline{h}_{14}\\ \underline{h}_{21} & \underline{h}_{22} & \underline{h}_{23} & \underline{h}_{24}\\ \underline{h}_{31} & \underline{h}_{32} & \underline{h}_{33} & \underline{h}_{34}\\ \underline{h}_{41} & \underline{h}_{42} & \underline{h}_{43} & \underline{h}_{44}\end{pmatrix}}_{\underline{\mathbf{H}}}\cdot\begin{pmatrix}\underline{F}_1\\ \underline{M}_1\\ \underline{F}_2\\ \underline{M}_2\end{pmatrix} \quad (4)$$

$$\begin{aligned}\underline{T}_{1,i}&=c^E_{11,i}\underline{S}_1-e_{31,i}\underline{E}_{3,i}\\ \underline{D}_{3,i}&=\varepsilon^T_{33,i}\underline{E}_{3,i}+d_{31,i}\underline{T}_{1,i}\end{aligned} \quad (5)$$

$$\begin{pmatrix}0\\ 0\\ \underline{v}_s\\ \underline{\Omega}_w\end{pmatrix}=\begin{pmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & \underline{h}_{33} & \underline{h}_{34}\\ 0 & 0 & \underline{h}_{43} & \underline{h}_{44}\end{pmatrix}\cdot\begin{pmatrix}\underline{F}_1\\ \underline{M}_1-\underline{M}_w\\ \underline{F}_s\\ -\underline{M}_w\end{pmatrix} \quad (6)$$

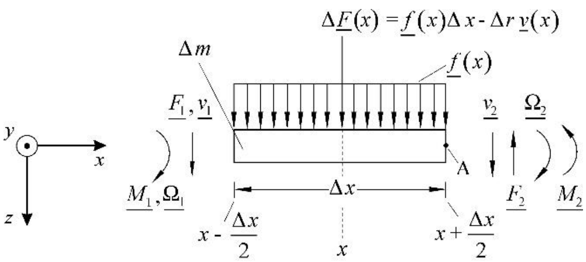

Fig. 1. Differential beam element.

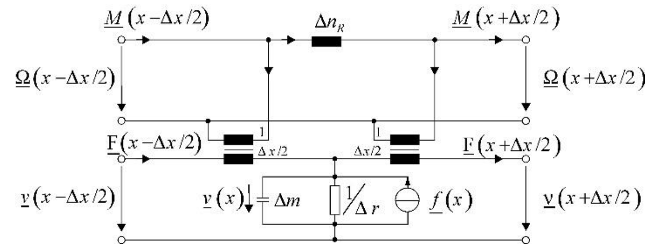

Fig. 2. Equivalent circuit representation of a differential beam element.

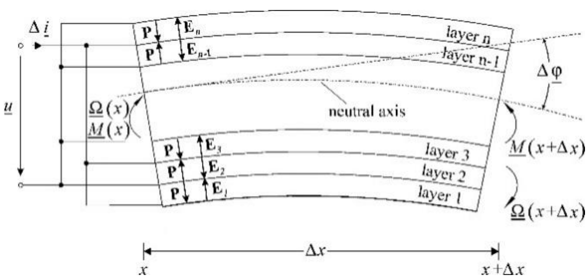

Fig. 3. Piezoceramic multilayered beam segment.

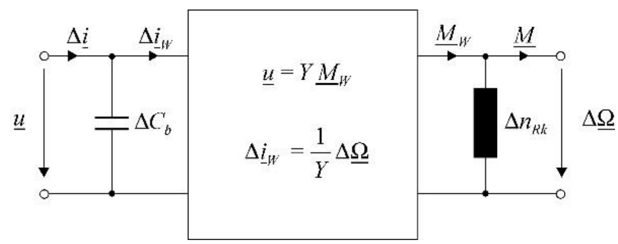

Fig. 4. Equivalent circuit representation of a multilayered piezoceramic beam segment.

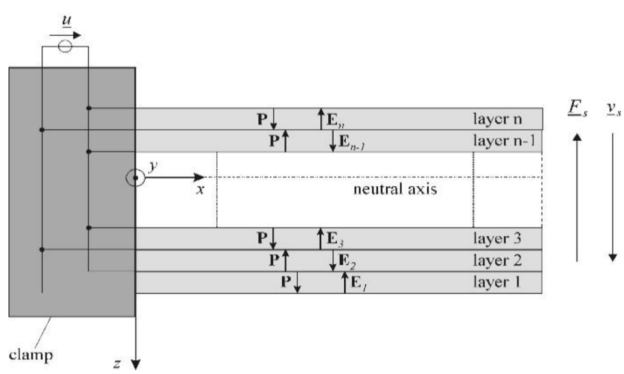

Fig. 5: Clamped-free piezoceramic multilayered transducer**.**

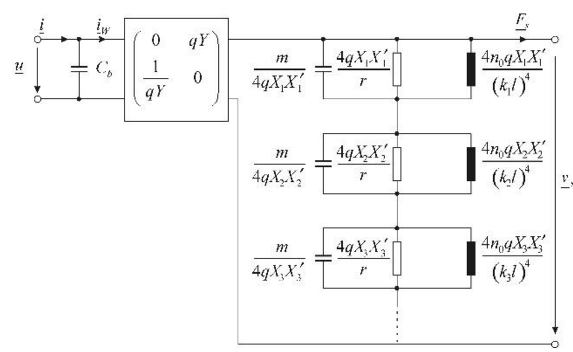

Fig. 6. Canonical equivalent circuit representation of a clamped-free piezoceramic multilayered transducer.

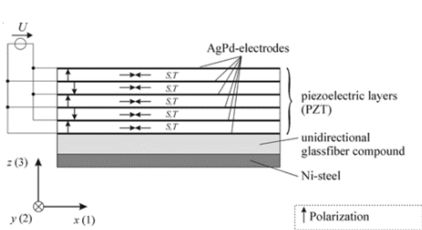

Fig. 7. Used piezoelectric actuator for experimental investigations.

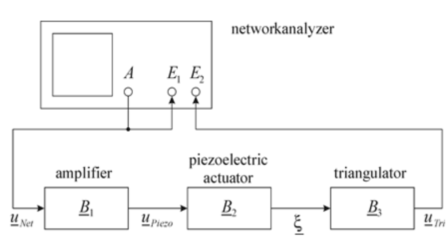

Fig. 8. Measurement setup for the experimental verification of the canonical equivalent circuit.

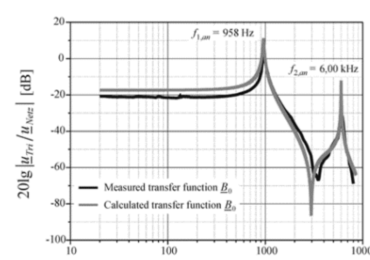

Fig. 9. Measured and calculated transfer function.

Abstract

In this paper the systematic derivation of the equivalent electromechanical circuit for any kind of clamped-free piezoceramic multilayered bending transducer with respect to Rayleigh's dissipation function is presented. In recent publications only bimorphs and triple layer piezoelectric benders have been represented by electrical equivalent circuits assuming a corresponding admittance matrix [**1-4**]. The aspects of the equivalent viscous damping in the sense of Rayleigh's dissipation function and the different vibration modes have not been taken into account yet.

Starting point of the consideration is a differential homogeneous beam element affected at both ends by as well complex-valued translatory quantities (velocity $\underline{v}$, force $\underline{F}$) as rotary quantities (angle velocity $\underline{\Omega}$, Moment $\underline{M}$) represents the starting point of considerations [**Fig. 1**]. In order to determine the dynamic behaviour of the beam element, the mass Δm, the equivalent viscous damping Δr and a continuous load $f(x)$ are taken into account. Assuming that the mass, the friction force and the load can be considered to be concentrated in the middle of the beam element, the inner structure of the differential beam element can be determined [**Fig. 2**]. This structure can also be described by the complex-valued partial differential equation for beam bending movements (eq. 1) where C, ω, μ and r_a represent the flexural rigidity, the angular frequency and the mass and friction per unit length. The solution of the differential equation can be described by an infinite series (eq. 2) where X_m(x) and $\underline{v}_m$ represent the displacement and velocity functions according to the m-th vibration mode [**5**]. The orthogonality properties of the displacement functions result in a general solution (eq. 3) which is the base for the determination of the dynamic admittance matrix $\underline{\mathbf{H}}$ (eq. 4).

In order to proceed to the piezoceramic transducer a multilayered beam segment with piezoelectric layers in bending deformation is considered [**Fig. 3**]. In combination with the piezoelectric constitutive equations (eq. 5) the connection between electrical and mechanical quantities can be described by an electromechanical circuit with two ports [**Fig. 4**] that can be implemented directly into to the equivalent circuit of a differential beam element [**Fig. 2**]. Assuming a piezoceramic multilayered cantilever [**Fig. 5**], the dynamic admittance matrix can be calculated (eq. 6) and the canonical electromechanical circuit representation of a clamped-free piezoceramic multilayered bending transducer can be derived [**Fig. 6**].

For the experimental investigations a specially developed piezoceramic multilayered bending transducer is used [**Fig. 7**]. In order to verify the canonical electromechanical circuit the emphasis is laid on the experimental determination of the dynamic deflection characteristics $\underline{\xi}$ of the bender's tip as a function of the driving voltage $\underline{u}$. The used measurement setup is shown in **Fig. 8**. The measurement results compared to the analytical calculations are shown in **Fig. 9**.

Keywords: Piezoceramic bending actuator; dynamic admittance matrix; canonical equivalent circuit representation; electromechanics; Rayleigh's dissipation function.

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Biography of Presenting Author

Prof. Dr.-Ing. Ruediger G. Ballas holds the professorship for electrical engineering at Wilhelm Buechner Hochschule Mobile University of Technology, Darmstadt, Germany. In addition to his activity as vice dean of the department of engineering sciences, his research activities focus in particularly on piezoelectric materials, sensors and actuators as well as on piezoelectric energy harvesting and (micro)electromechanical systems (MEMS). Before his university career he held leading positions in research and development at different companies concerned with piezoelectric sensors and actuators. He earned his doctorate degree at Technical University Darmstadt, Germany for his research work in the field of piezoelectric multi-layered bending actuators. During his pre-doctoral studies, he already received several awards for outstanding work in the field of micro structuring of quartz crystals for high-frequency applications in modern telecommunications technologies. Furthermore, in 2017 he received the IAAM Medal due to notable and outstanding contribution in the field of ”New Age Technology & Innovations”, Stockholm, Sweden. In 2020 he was nominated as ”Fellow of the International Association of Advanced Materials (FIAAM), Sweden” as well as ”VSET Fellow (Vebleo - International Scientific Organization for Materials Science, Engineering and Technology), Sweden”. He is author of various publications and author/co-author of several textbooks published by the renowned publisher Springer Nature.

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